

New Approach of Solving Quadratic Equation

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ABSTRACT: The objective of this present work is to introduce a new method of solving quadratic equations different from the existing methods. In this paper, we explicitly explain the new method and carefully discussed the conditions necessary for using this new approach. Finally, we apply our new approach to solve some questions and compare the result with the existing methods and results coincide. **KEYWORD:** The following are the keywords to be discuss namely; Affected quadratic equation, Absolute value and real number.

METHOD # 1:

Here, we consider the equation of the form below;
 $ax^{n+1} + bx^n + nc = 0 \dots\dots\dots(1) \quad \forall n \in \mathbb{R}, n = 0, 1$

by putting $n = 1$ we have;
 $ax^2 + bx^1 + c = 0 \dots\dots\dots(2)$

from equation (1), divide through by nc

$$\left(\frac{a}{nc}\right)x^n \cdot x + \left(\frac{b}{nc}\right)x^n + 1 = 0$$

Multiply through by x^{-n}

$$x^{-n} + \left(\frac{a}{nc}\right)x + \left(\frac{b}{nc}\right) = 0$$

by setting $IAI = \frac{a}{nc}$ and $IBI = \frac{b}{nc}$

$$x^{-n} + IAIx^1 + IBI = 0 \dots\dots\dots(3)$$

Also, from equation (3), multiply through by x^{-1} we have

$$x^{-(n+1)} + IBIx^{-1} + IAI = 0 \dots\dots\dots(4)$$

$$x^{-(n+1)} + IBIx^{-1} = -IAI$$

Square both sides and simplify, we obtain

$$(x^{-n} + IBI)(x^{-n} + IBI) = (IAIx^1)^2$$

Square both sides we have;

$$x^{-n} - IAIx + IBI = 0 \dots\dots\dots(5)$$

Since, we define that;

$$IAI = \begin{cases} A, A \geq 0 \\ -A, A \leq 0 \end{cases} \quad \text{and} \quad IBI = \begin{cases} B, B \geq 0 \\ -B, B \leq 0 \end{cases}$$

Taking $IAI = A$ and $IBI = -B$

Hence, we have;

$$x^{-n} - Ax = B \dots\dots\dots(6)$$

Divide through by B

$$(Bx^n)^{-1} - AB^{-1}x = 1$$

Setting $P = Bx^n$ and $R = AB^{-1}$

$$P^{-1} - Rx = 1$$

On simplifying we have;

$$X = \frac{1}{R} (P^{-1} - 1) \dots\dots\dots(7)$$

Where equation (7) gives real solution of equation (2), hence the method is applicable for $b^2 \geq 4ac$. So, from equation (7) by merely looking, the roots of the quadratic equation are obtained as follow

$$x_\alpha = \frac{\mu}{\Omega} \quad \text{and} \quad x_\beta = \eta - x_\alpha \dots\dots\dots(8)$$

where μ is the coefficient of x^{-1} obtained from equation (7) and η denoted $-\frac{1}{R}$ from equation (7) and Ω is obtain using the following procedure

- (1) When the coefficient of x^{-1} is positive and all the signs of the quadratic equation are positive then the value of the coefficient of x^{-1} should be negative and vice-versa.
- (2) If any of coefficient of x^{-1} or other has denominator of 2,3,4,5 and so on and the other do not have, express it as the denominator of above mentioned numbers
- (3) At $a \neq 1$, if the coefficient of b is positive the first root must be positive and vice-versa.
- (4) One of the roots of quadratic equation must have a(coefficient of x^2) as the denominator.

METHOD # 2:

In this method, we assume the complex solution of the form below;

$$X = m + ni \dots\dots\dots(9)$$

$$\text{So, } x^2 = m^2 - n^2 + 2mni$$

$$\text{Since, } ax^2 + bx + c = 0$$

Divide through by a

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\text{Hence, } (m^2 - n^2 + 2mni) + \frac{b}{a}(m + ni) + \frac{c}{a} = 0$$

$$(m^2 - n^2) + 2mni + \frac{b}{a}m + \frac{b}{a}ni + \frac{c}{a} = 0$$

$$\left((m^2 - n^2) + \frac{b}{a}m + \frac{c}{a} \right) + \left(2mn + \frac{b}{a}n \right)i = 0$$

By comparing the coefficient we have;

$$m^2 - n^2 + \frac{b}{a}m + \frac{c}{a} = 0 \dots\dots\dots(i)$$

$$2mn + \frac{b}{a}n =$$

$$0 \dots\dots\dots(ii)$$

From equation (ii)

$$m = -\frac{b}{2a}$$

$$\dots\dots\dots(iii)$$

Where m is substituted into equation (i) to obtain the value of n, hence; the solution to any given quadratic equation is obtained.

EXAMPLE 1: Given the quadratic equation below

$$x^2 - 7x + 10 = 0$$

We can solve the above equation using both methods

METHOD # 2:

$$x = m + ni \quad m^2 - n^2 + 2mni - 7m -$$

$$7ni = -10$$

$$\text{therefore, } m^2 - n^2 - 7m = -10 \dots\dots\dots(i)$$

$$\text{and } 2mn = 0 \dots\dots\dots(ii)$$

From equation (ii)

$$m = \frac{7}{2}$$

On substituting the value of m into equation (i)

$$\left(\frac{7}{2}\right)^2 - n^2 - 7\left(\frac{7}{2}\right) = -10$$

On evaluating we have;

$$n^2 = -\frac{9}{4} \quad \text{hence} \quad n = \pm \frac{3}{2}i$$

$$\text{Now, } x = m + ni = \frac{7}{2} \pm \frac{3}{2}i^2 = \frac{7}{2} - \frac{3}{2}$$

$$x = \frac{7}{2} - \frac{3}{2} = 2 \quad \text{OR} \quad x = \frac{7}{2} + \frac{3}{2} = 5$$

Hence, the solution to the above quadratic equation are 2 and 5.

Similarly, the solution to the given quadratic equation is obtained by using the first method but the procedures mentioned above must follow.